Force System Resultants

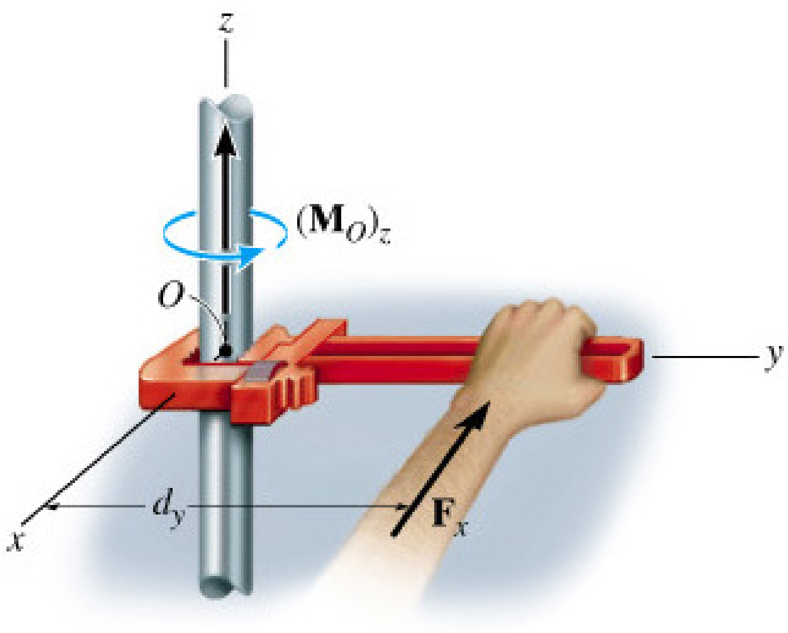
Moments, Couples, and Force Couple Systems

Equivalent Forces

- We defined equivalent forces as being forces with the same magnitude acting in the same direction and acting along the same line of action (this is through the Principle of Transmissibility), but why do the forces need to act along the same line?

4.1 Introduction to Moments

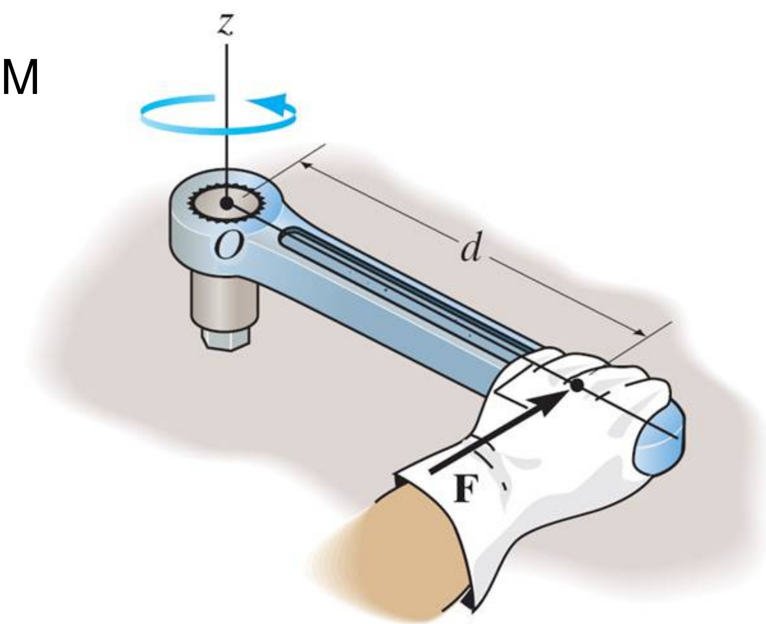
The tendency of a force to rotate a rigid body about any defined axis is called the Moment of the force about the axis



(a)

MOMENT OF A FORCE - SCALAR FORMULATION (Section 4.1)

M



The moment, M, of a force about a point provides a measure of the tendency for rotation (sometimes called a torque).

$$M = F * d$$

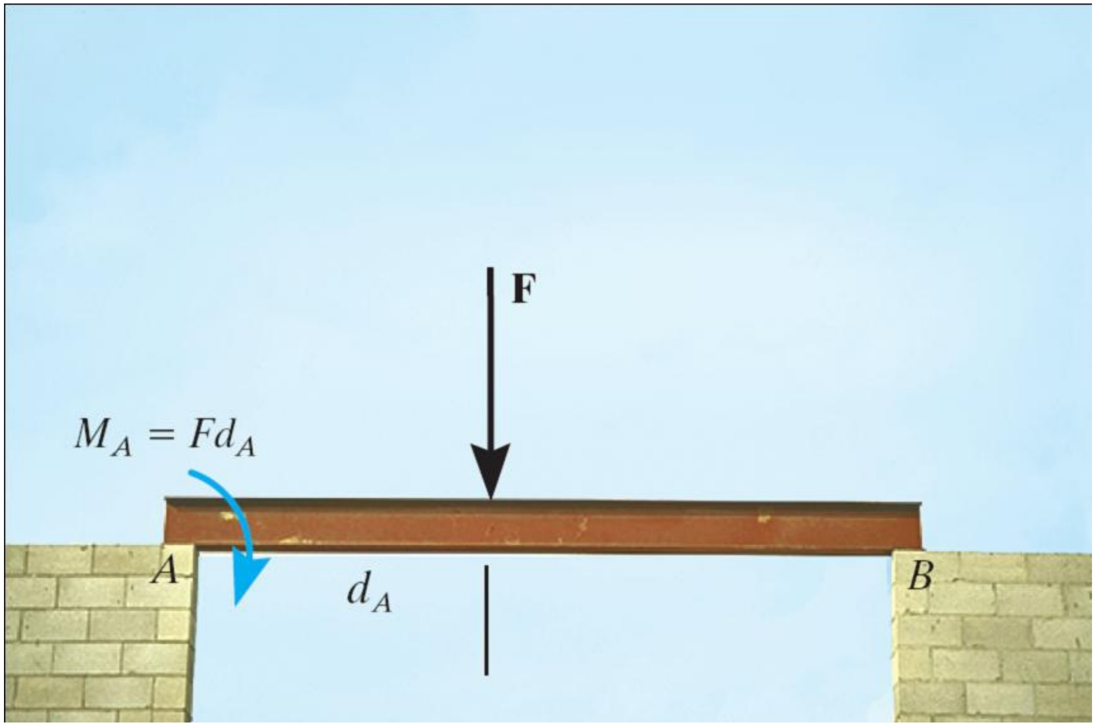
Moment caused by a Force

- The Moment of Force (**F**) about an axis through Point (A) or for short, the Moment of **F** about A, is the product of the magnitude of the force and the perpendicular distance between Point (A) and the line of action of Force (**F**)
  - $M_A = Fd$

Units of a Moment

- The units of a Moment are:
  - *N·m* in the SI system
  - *ft·lbs* or *in·lbs* in the US Customary system

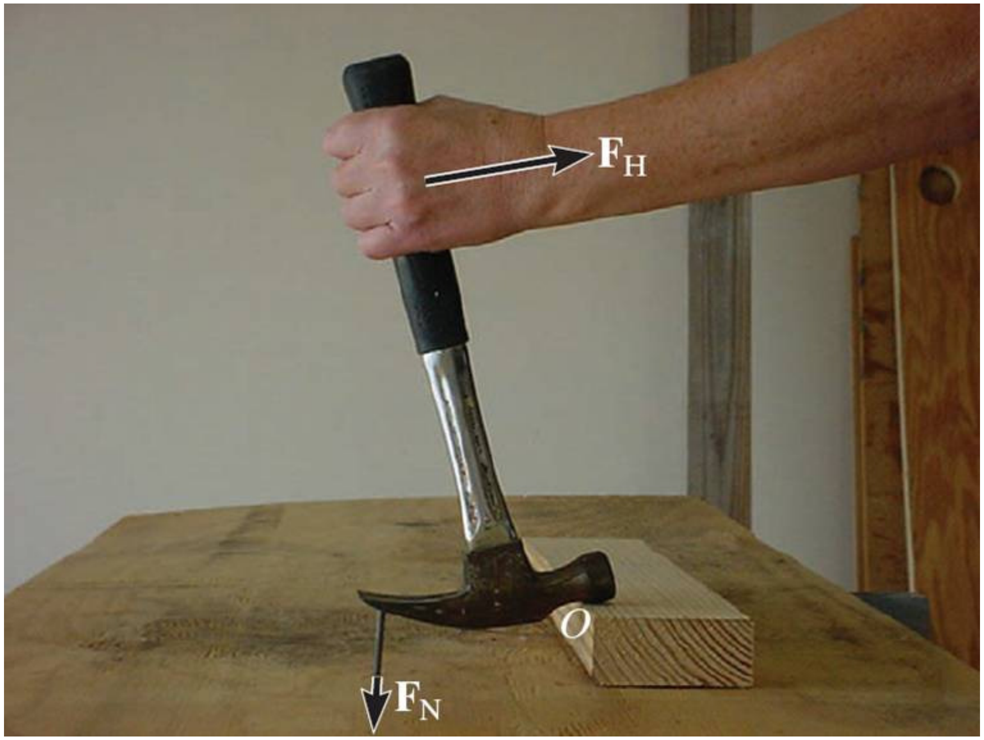
APPLICATIONS



Beams are often used to bridge gaps in walls. We have to know what the effect of the force on the beam will have on the beam supports.

What do you think those impacts are at points A and B?

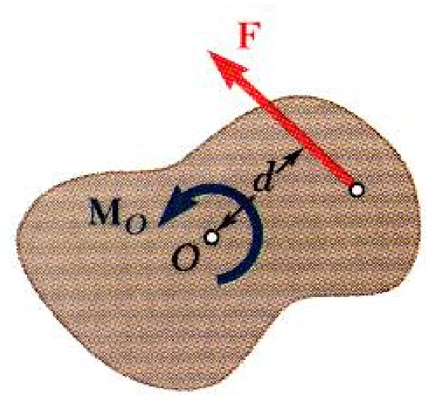
APPLICATIONS



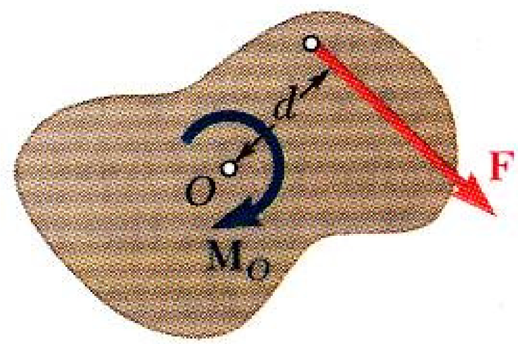
Carpenters often use a hammer in this way to pull a stubborn nail. Through what sort of action does the force F_H at the handle pull the nail? How can you mathematically model the effect of force F_H at point O?

Properties of a Moment

- Moments not only have a magnitude, they also have a sense to them.
- The sense of a moment is clockwise or counter-clockwise depending on which way it will tend to make the object rotate



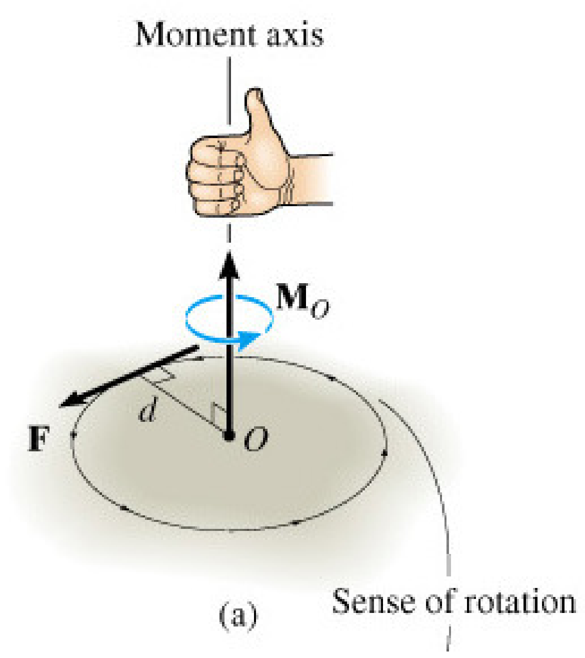
(a) $M_O = +Fd$



(b) $M_O = -Fd$

Properties of a Moment

- The sense of a Moment is defined by the direction it is acting on the Axis and can be found using Right Hand Rule.



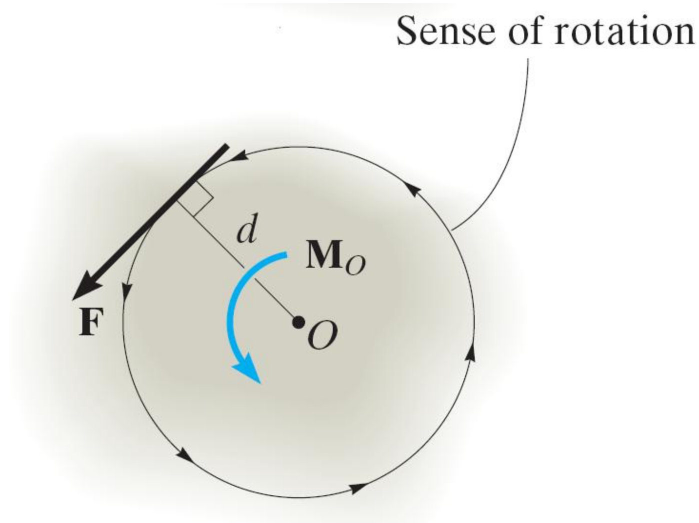
(a)

Varignon's Theorem

- *The moment of a Force about any axis is equal to the sum of the moments of its components about that axis*
- This means that resolving or replacing forces with their resultant force will not affect the moment on the object being analyzed

MOMENT OF A FORCE - SCALAR FORMULATION (continued)

In the 2-D case, the magnitude of the moment is $M_o = F\ d$



As shown, d is the perpendicular distance from point O to the line of action of the force.

In 2-D, the direction of M_O is either clockwise or

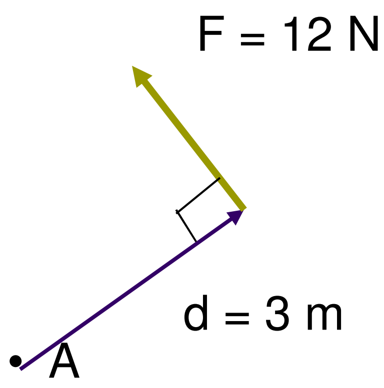
counter-clockwise, depending on the tendency for rotation.

READING QUIZ

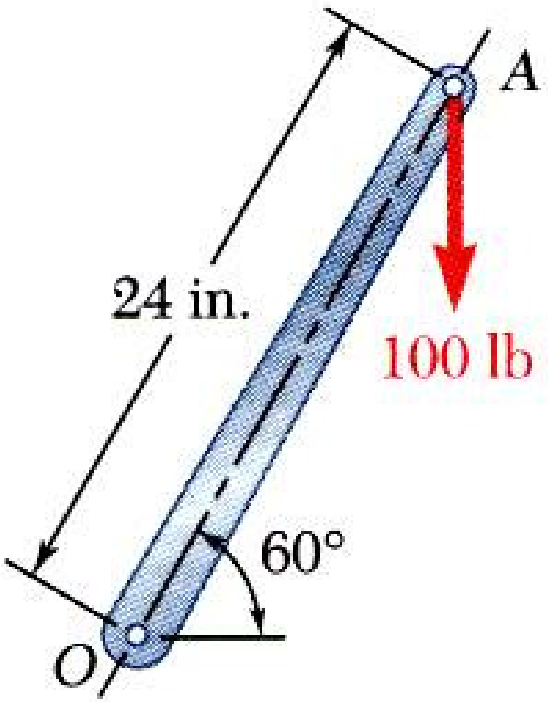
1. What is the moment of the 10 N force about point A (M_A)?

   A) 3 N·m   B) 36 N·m   C) 12 N·m

   D) (12/3) N·m   E) 7 N·m

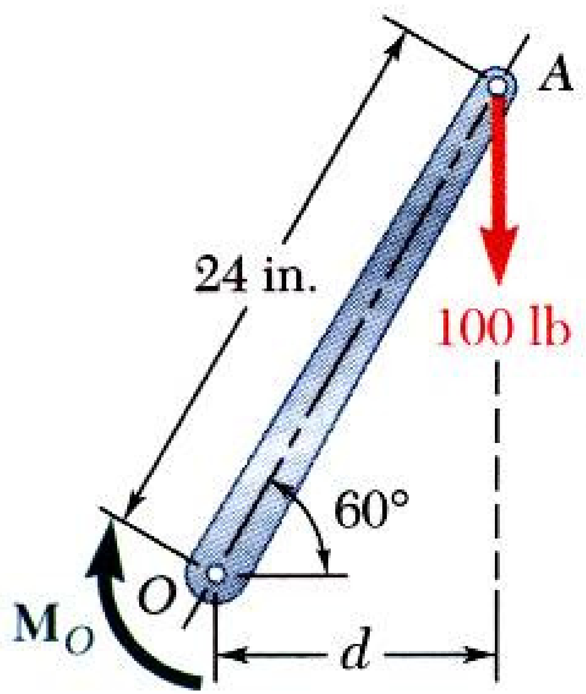


Example #1



- A 100-lb vertical force is applied to the end of a lever which is attached to a shaft at *O*.
- Determine:
  a) Moment about *O,*
  b) Horizontal force at *A* which creates the same moment,
  c) Smallest force at A which produces the same moment,
  d) Location for a 240-lb vertical force to produce the same moment,
  e) Whether any of the forces from b, c, and d is equivalent to the original force.

Example #1



a) Moment about O is equal to the product of the force and the perpendicular distance between the line of action of the force and O. Since the force tends to rotate the lever clockwise, the moment vector is into the plane of the paper.

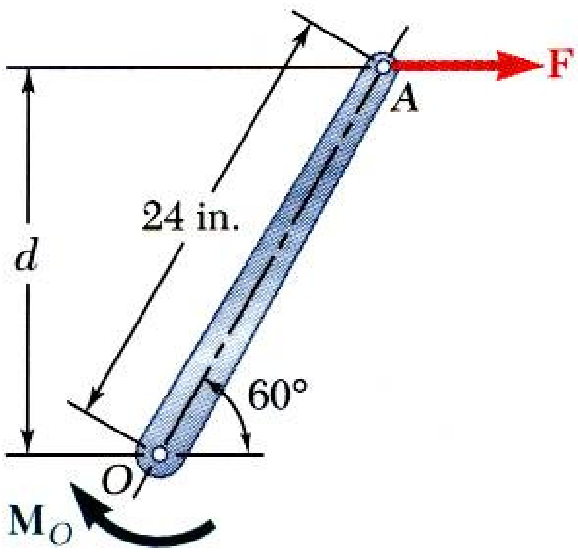
$$M_O = Fd$$

$$d = (24\,\text{in.})\cos 60° = 12\,\text{in.}$$

$$M_O = (100\,\text{lb})(12\,\text{in.})$$

$$\boxed{M_O = 1200\,\text{lb}\cdot\text{in}}$$

Example #1



b) Horizontal force at A that produces the same moment,

$$d = (24\text{ in.})\sin 60° = 20.8\text{ in.}$$

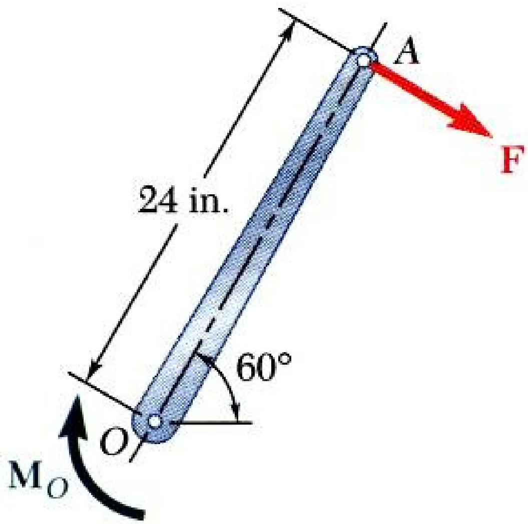
$$M_O = Fd$$

$$1200\text{ lb}\cdot\text{in.} = F(20.8\text{ in.})$$

$$F = \frac{1200\text{ lb}\cdot\text{in.}}{20.8\text{ in.}}$$

$$\boxed{F = 57.7\text{ lb}}$$

Example #1



c) The smallest force at *A* to produce the same moment occurs when the perpendicular distance is a maximum or when *F* is perpendicular to *OA*.

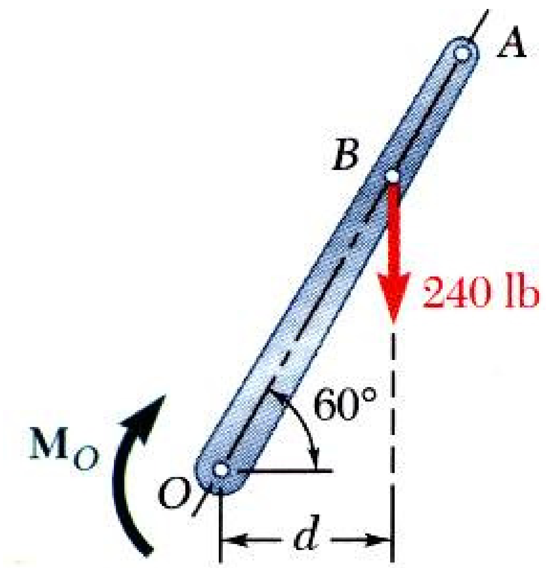
$$M_O = Fd$$

$$1200\,\text{lb}\cdot\text{in.} = F(24\,\text{in.})$$

$$F = \frac{1200\,\text{lb}\cdot\text{in.}}{24\,\text{in.}}$$

$$\boxed{F = 50\,\text{lb}}$$

Example #1



d) To determine the point of application of a 240 lb force to produce the same moment,

$$M_O = Fd$$

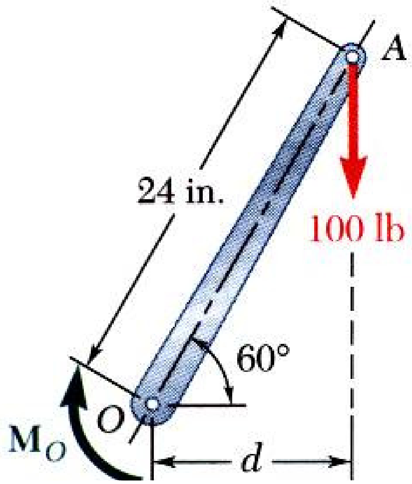
$$1200\,\text{lb}\cdot\text{in.} = (240\,\text{lb})d$$

$$d = \frac{1200\,\text{lb}\cdot\text{in.}}{240\,\text{lb}} = 5\,\text{in.}$$

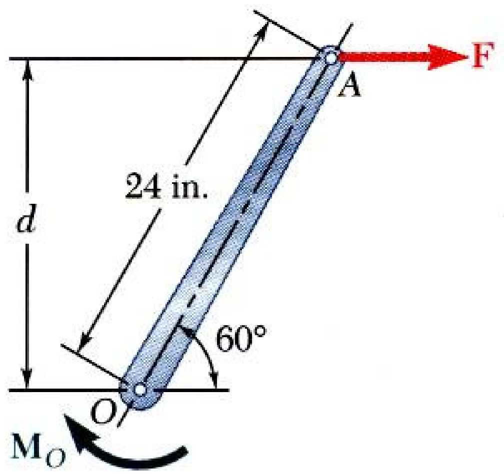
$$OB\cos 60° = 5\,\text{in.}$$

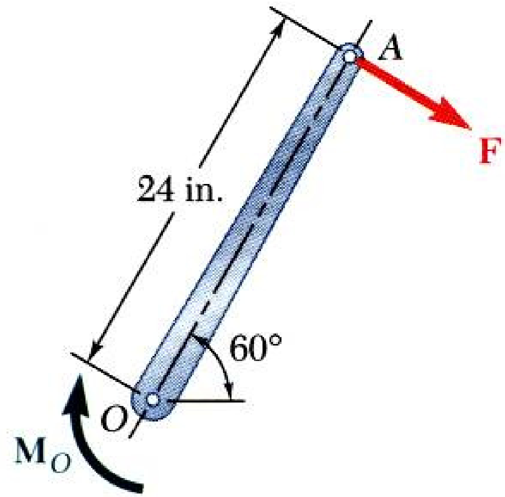
$$\boxed{OB = 10\,\text{in.}}$$

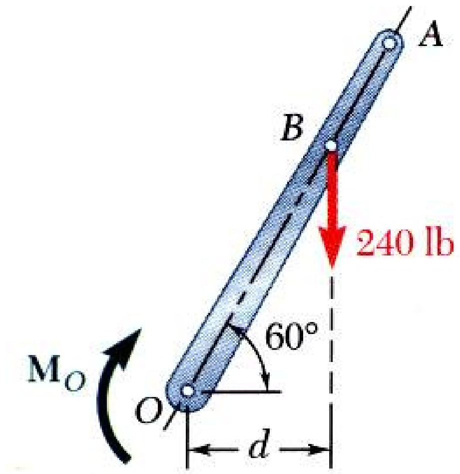
Example #1



e) Although each of the forces in parts b), c), and d) produces the same moment as the 100 lb force, none are of the same magnitude and sense, or on the same line of action. None of the forces is equivalent to the 100 lb force.







EXAMPLE 4.1

For each case illustrated in Fig. 4–4, determine the moment of the force about point O.

SOLUTION *(SCALAR ANALYSIS)*

The line of action of each force is extended as a dashed line in order to establish the moment arm d. Also illustrated is the tendency of rotation of the member as caused by the force. Furthermore, the orbit of the force about O is shown as a colored curl. Thus,

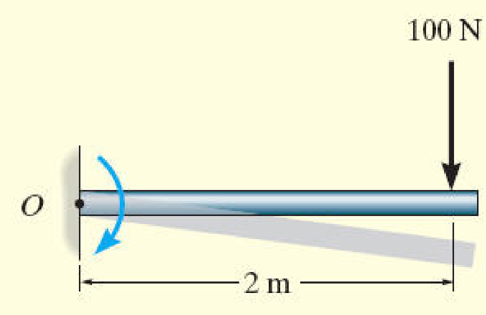
Fig. 4–4*a* $\quad M_O = (100\text{ N})(2\text{ m}) = 200\text{ N}\cdot\text{m} \circlearrowright$ *Ans.*

Fig. 4–4*b* $\quad M_O = (50\text{ N})(0.75\text{ m}) = 37.5\text{ N}\cdot\text{m} \circlearrowright$ *Ans.*

Fig. 4–4*c* $\quad M_O = (40\text{ lb})(4\text{ ft} + 2\cos 30^\circ\text{ ft}) = 229\text{ lb}\cdot\text{ft} \circlearrowright$ *Ans.*

Fig. 4–4*d* $\quad M_O = (60\text{ lb})(1\sin 45^\circ\text{ ft}) = 42.4\text{ lb}\cdot\text{ft} \circlearrowleft$ *Ans.*

Fig. 4–4*e* $\quad M_O = (7\text{ kN})(4\text{ m} - 1\text{ m}) = 21.0\text{ kN}\cdot\text{m} \circlearrowleft$ *Ans.*



(a)

EXAMPLE 4.1 CONTINUED

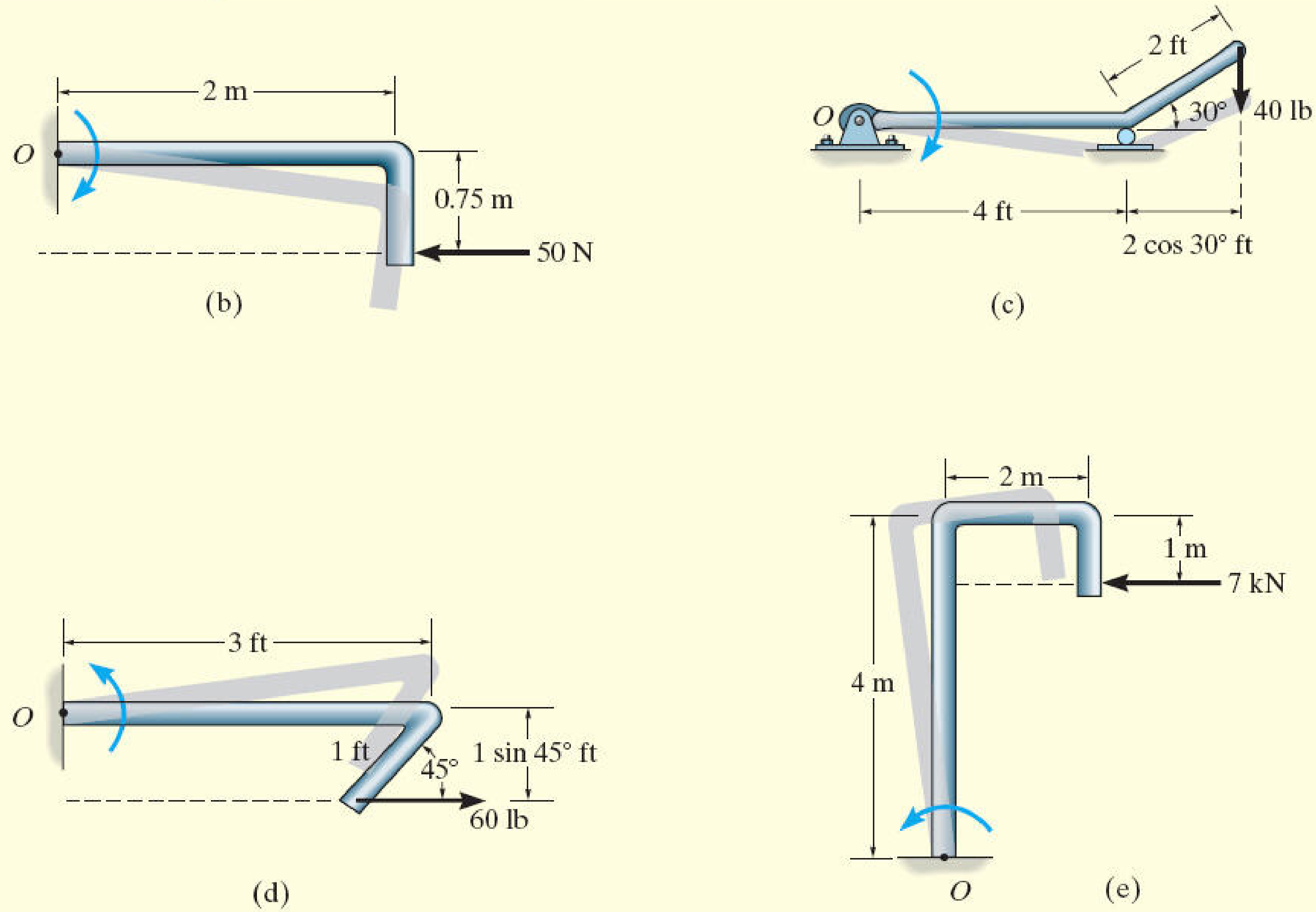


Fig. 4–4

EXAMPLE 4.2

Determine the resultant moment of the four forces acting on the rod shown in Fig. 4–5 about point O.

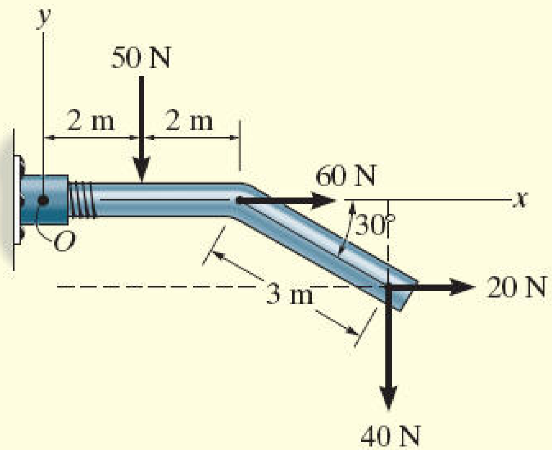


Fig. 4–5

SOLUTION

Assuming that positive moments act in the $+\mathbf{k}$ direction, i.e., counterclockwise, we have

$$\circlearrowleft + M_{R_O} = \Sigma Fd;$$

$$M_{R_O} = -50\text{ N}(2\text{ m}) + 60\text{ N}(0) + 20\text{ N}(3\sin 30^\circ\text{ m})$$

$$-40\text{ N}(4\text{ m} + 3\cos 30^\circ\text{ m})$$

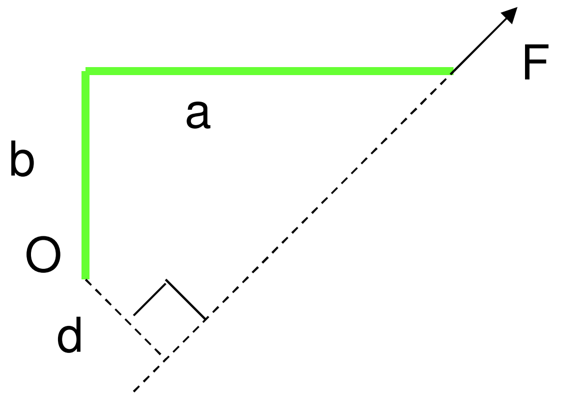
$$M_{R_O} = -334\text{ N}\cdot\text{m} = 334\text{ N}\cdot\text{m}\circlearrowright \qquad \textit{Ans.}$$

For this calculation, note how the moment-arm distances for the 20-N and 40-N forces are established from the extended (dashed) lines of action of each of these forces.

4.4 Principle of Moments

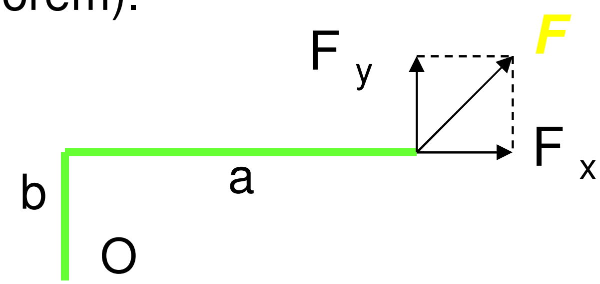
- Varignon's Theorem: The moment of a force about a point is equal to the sum of moments of the components of the force about the point:

MOMENT OF A FORCE - SCALAR FORMULATION (continued)



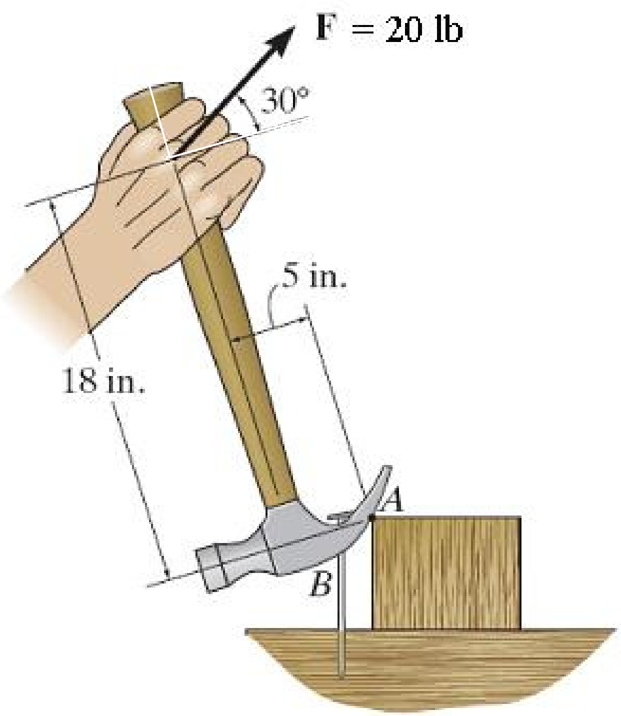
For example, $M_O = F\,d$ and the direction is counter-clockwise.

Often it is easier to determine M_O by using the components of ***F*** as shown (Varignon's Theorem).



Then $M_O = (F_Y\, a) – (F_X\, b)$. Note the different signs on the terms! The typical sign convention for a moment in 2-D is that counter-clockwise is considered positive. We can determine the direction of rotation by imagining the body pinned at O and deciding which way the body would rotate because of the force.

GROUP PROBLEM SOLVING



Given: A 20 lb force is applied to the hammer.

Find: The moment of the force at A.

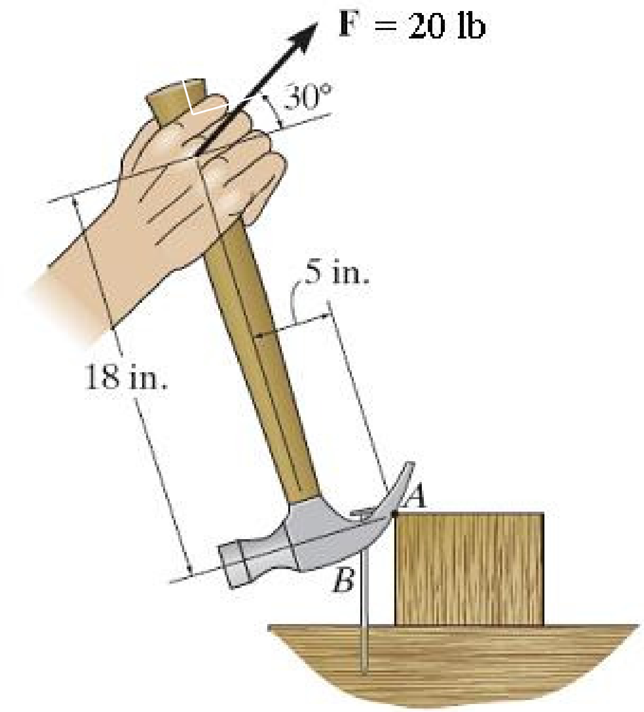
Plan:

Since this is a 2-D problem:

1) Resolve the 20 lb force along the handle's x and y axes.

2) Determine M_A using a scalar analysis.

GROUP PROBLEM SOLVING (cont.)



Solution:

$+\uparrow F_y = 20 \sin 30° \text{ lb}$

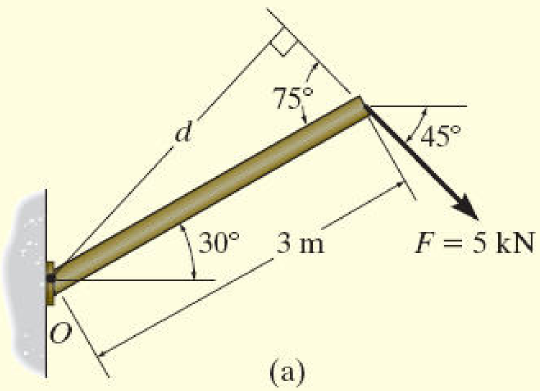
$+\rightarrow F_x = 20 \cos 30° \text{ lb}$

$+\circlearrowleft M_A = \{-(20 \cos 30°)\text{lb } (18 \text{ in}) - (20 \sin 20°)\text{lb } (5 \text{ in})\}$

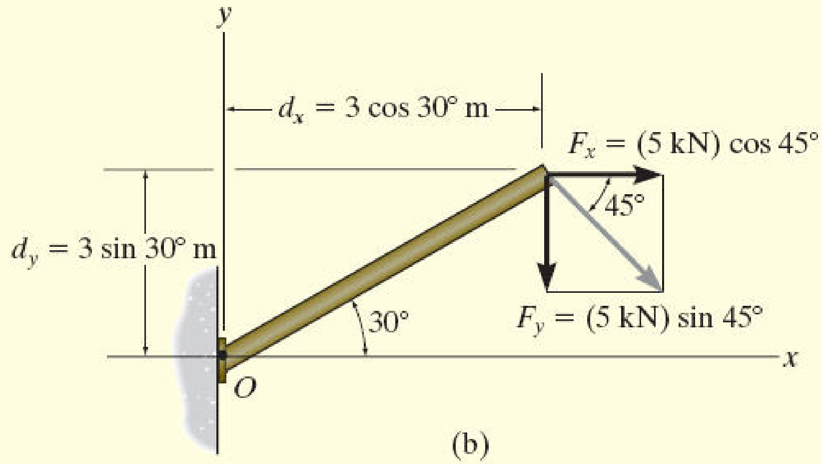
$= -351.77 \text{ lb·in} = 352 \text{ lb·in (clockwise)}$

EXAMPLE 4.5

Determine the moment of the force in Fig. 4–18*a* about point *O*.



(a)



(b)

SOLUTION I

The moment arm d in Fig. 4–18*a* can be found from trigonometry.

$$d = (3\text{ m})\sin 75^\circ = 2.898\text{ m}$$

Thus,

$$M_O = Fd = (5\text{kN})(2.898\text{ m}) = 14.5\text{ kN}\cdot\text{m} \circlearrowright \qquad \textit{Ans.}$$

Since the force tends to rotate or orbit clockwise about point O, the moment is directed into the page.

EXAMPLE 4.5 CONTINUED

SOLUTION II

The x and y components of the force are indicated in Fig. 4–18b. Considering counterclockwise moments as positive, and applying the principle of moments, we have

$$\circlearrowleft + M_O = -F_x d_y - F_y d_x$$

$$= -(5 \cos 45^\circ \text{ kN})(3 \sin 30^\circ \text{ m}) - (5 \sin 45^\circ \text{ kN})(3 \cos 30^\circ \text{ m})$$

$$= -14.5 \text{ kN} \cdot \text{m} = 14.5 \text{ kN} \cdot \text{m} \circlearrowright \qquad \textit{Ans.}$$

SOLUTION III

The x and y axes can be set parallel and perpendicular to the rod's axis as shown in Fig. 4-18c. Here $\mathbf{F}_x$ produces no moment about point O since its line of action passes through this point. Therefore,

$$\circlearrowleft + M_O = -F_y d_x$$

$$= -(5 \sin 75^\circ \text{ kN})(3 \text{ m})$$

$$= -14.5 \text{ kN} \cdot \text{m} = 14.5 \text{ kN} \cdot \text{m} \circlearrowright \qquad \textit{Ans.}$$

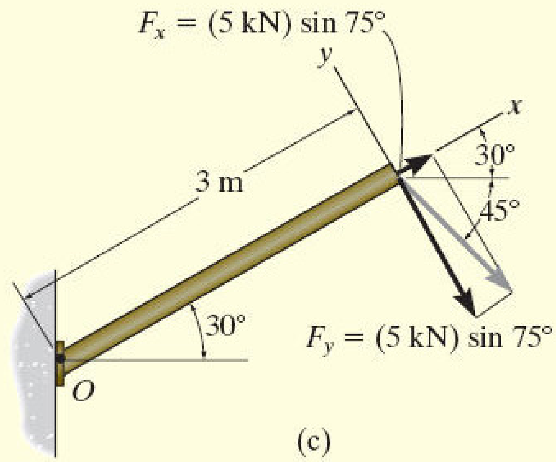


Fig. 4–18

Moments in 3D
4.5 Moment of a Force about a Specific Axis

- In 2D bodies the moment is due to a force contained in the plane of action perpendicular to the axis it is acting around. This makes the analysis very easy.
- In 3D situations, this is very seldom found to be the case.

Moments in 3D

- The moment about an axis is still calculated the same way (by a force in the plane perpendicular to the axis) but most forces are acting in abstract angles.
- By resolving the abstract force into its rectangular components (or into its components perpendicular to the axis of concern) the moment about the axis can then be found the same way it was found in 2D – M = Fd (where d is the distance between the force and the axis of concern)

Notation for Moments

- In simpler terms the Moment of a Force about the *y*-axis (M_y) can be found by using the projection of the Force on the *x*-*z* Plane

- The Notation used to denote Moments about the Cartesian Axes are (M_x, M_y, and M_z)

EXAMPLE 4.7

Determine the resultant moment of the three forces in Fig. 4–22 about the x axis, the y axis, and the z axis.

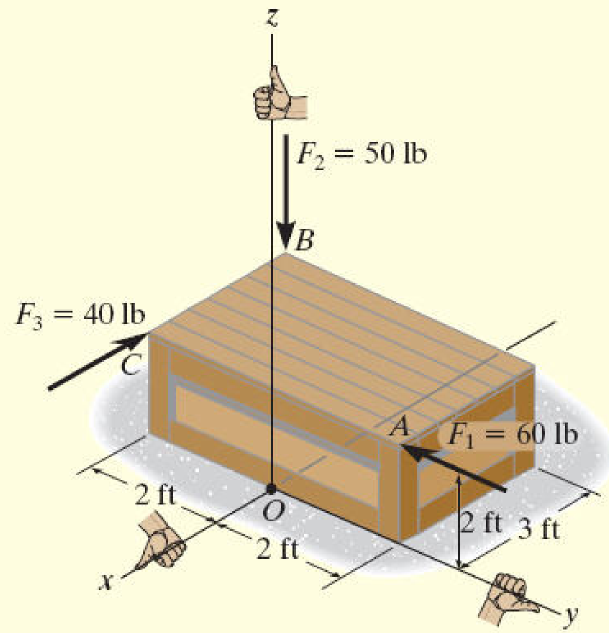


Fig. 4–22

SOLUTION

A force that is *parallel* to a coordinate axis or has a line of action that passes through the axis does *not* produce any moment or tendency for turning about that axis. Therefore, defining the positive direction of the moment of a force according to the right-hand rule, as shown in the figure, we have

$$M_x = (60 \text{ lb})(2 \text{ ft}) + (50 \text{ lb})(2 \text{ ft}) + 0 = 220 \text{ lb} \cdot \text{ft} \qquad \textit{Ans.}$$

$$M_y = 0 - (50 \text{ lb})(3 \text{ ft}) - (40 \text{ lb})(2 \text{ ft}) = -230 \text{ lb} \cdot \text{ft} \qquad \textit{Ans.}$$

$$M_z = 0 + 0 - (40 \text{ lb})(2 \text{ ft}) = -80 \text{ lb} \cdot \text{ft} \qquad \textit{Ans.}$$

The negative signs indicate that $\mathbf{M}_y$ and $\mathbf{M}_z$ act in the $-y$ and $-z$ directions, respectively.

3D Moment:

Example 4–17

Determine the magnitude of the moment about point O of the three components of the force $\mathbf{F}$ acting at the end A of the pipe in Fig. 4–34a. Specify the coordinate direction angles of the moment axis of the resultant moment.

Solution

The coordinate or perpendicular distances from the line of action of the force components to the x, y, z axes are indicated in the figure. Using the right-hand rule, we will consider each moment component separately.

For $\mathbf{M}_x$: Note that $\mathbf{F}_x$ does not create a moment about the x axis since $\mathbf{F}_x$ is parallel to the x axis. Also, $\mathbf{F}_y$ does not create a moment about the x axis since the line of action of $\mathbf{F}_y$ passes through a point lying on the axis. The moment of $\mathbf{F}_z$ about the x axis is $-(10 \text{ lb})(3 \text{ ft})$. Thus,

$$M_x = 0 + 0 - (10 \text{ lb})(3 \text{ ft}) = -30 \text{ lb} \cdot \text{ft}$$

For $\mathbf{M}_y$: $\mathbf{F}_x$ and $\mathbf{F}_y$ do not create moments about the y axis. Why? The moment of $\mathbf{F}_z$ about the y axis is $+(10 \text{ lb})(2 \text{ ft})$. Thus

$$M_y = 0 + 0 + (10 \text{ lb})(2 \text{ ft}) = 20 \text{ lb} \cdot \text{ft}$$

For $\mathbf{M}_z$: Here $\mathbf{F}_x$ creates a moment about the z axis of $-(4 \text{ lb})(3 \text{ ft})$ and $\mathbf{F}_y$ creates a moment of $+(12 \text{ lb})(2 \text{ ft})$. $\mathbf{F}_z$ does not contribute moment about the z axis. Thus,

$$M_z = -(4 \text{ lb})(3 \text{ ft}) + (12 \text{ lb})(2 \text{ ft}) + 0 = 12 \text{ lb} \cdot \text{ft}$$

Using Eq. 4–6, the magnitude of the resultant moment at O is thus

$$M_O = \sqrt{(-30)^2 + (20)^2 + (12)^2} = 38.0 \text{ lb} \cdot \text{ft} \qquad \textit{Ans.}$$

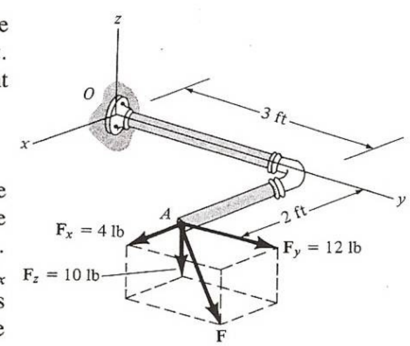
The coordinate direction angles of the moment axis are determined from Eqs. 4–7, i.e.,

$$\alpha = \cos^{-1}\left(\frac{-30}{38.0}\right) = 142.1^\circ \qquad \textit{Ans.}$$

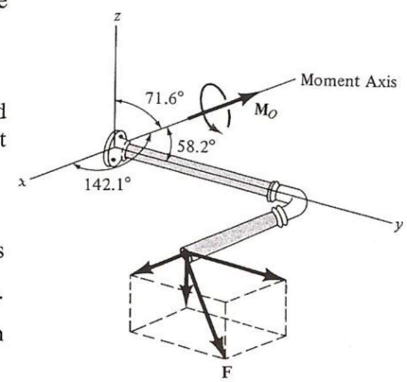
$$\beta = \cos^{-1}\left(\frac{20}{38.0}\right) = 58.2^\circ \qquad \textit{Ans.}$$

$$\gamma = \cos^{-1}\left(\frac{12}{38.0}\right) = 71.6^\circ \qquad \textit{Ans.}$$

The results are shown in Fig. 4–34b. Realize that when the moment M_O of a force is to be determined about a point, this actually implies finding the moment of the force about a *point lying on the moment axis*.



(a)



(b)

Fig. 4–34

3D Moments Example:

•Given the tension in cable BC is 700 N, find Mx, My, and Mz about point A.